Numerical Solution to the Performability of a Multiprocessor System with Reconfiguration and Rebooting Delays

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Outline

- Introduction
- Multiprocessor system with identical processors
- Modelling reconfiguration and rebooting delays in multiprocessor systems
- The steady state solution
- Numerical Results
- Conclusions

Introduction

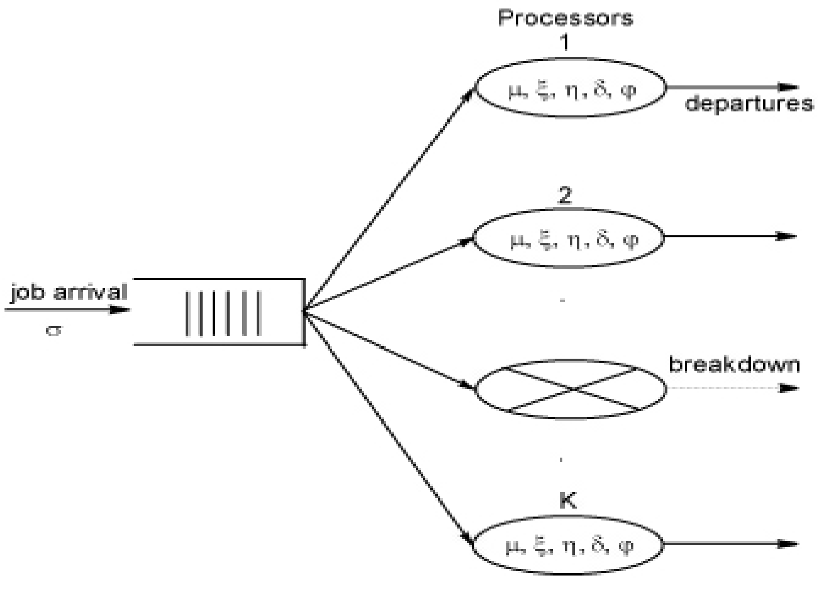
- Multi-server system models are useful to model
  - multiprocessor systems
  - nodes in communication networks, and
  - flexible machine shops
- approaches to model homogeneous multiprocessor systems with reconfiguration and rebooting delays

Introduction

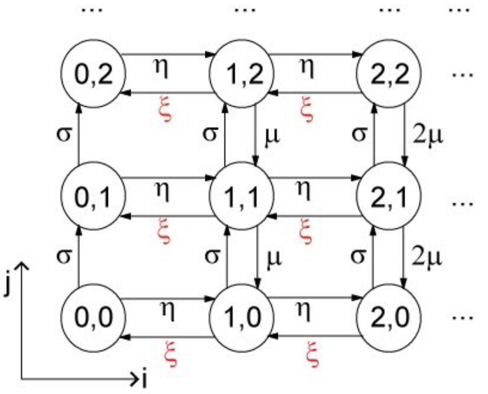
- an exact solution for the steady state probabilities of this problem using the spectral expansion method.
- The effects of reconfiguration and rebooting delays are analysed

Multiprocessor system with identical processors

- A homogeneous multiprocessor system, with K identical parallel processors, and a common queue is considered.
- The queue is of capacity L (finite or infinite $L \geq K$), including the jobs in service.
- Jobs arrive into the system in a Poisson stream at rate σ, and join the queue.
- Jobs are homogeneous and the service rates of the processors assumed identical.
  - the service times of jobs serviced by processor k (k=1, 2, …, K) are distributed exponentially with mean $1/\mu$.



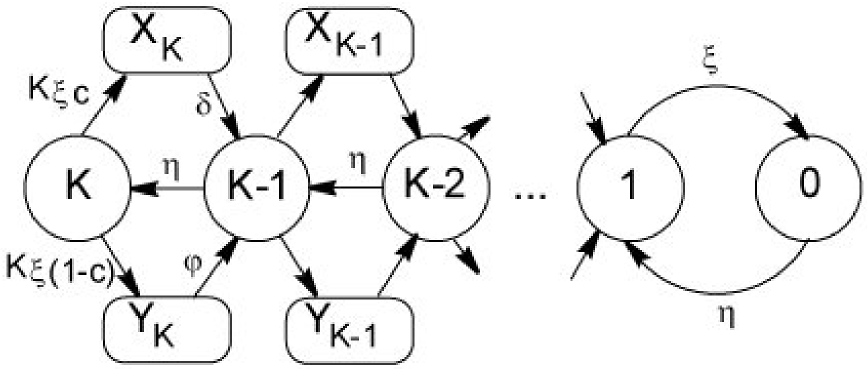
States and transitions



i: operative state (Number of operational processors)
j: number of jobs in the system

Modelling reconfiguration and rebooting delays in multiprocessor systems

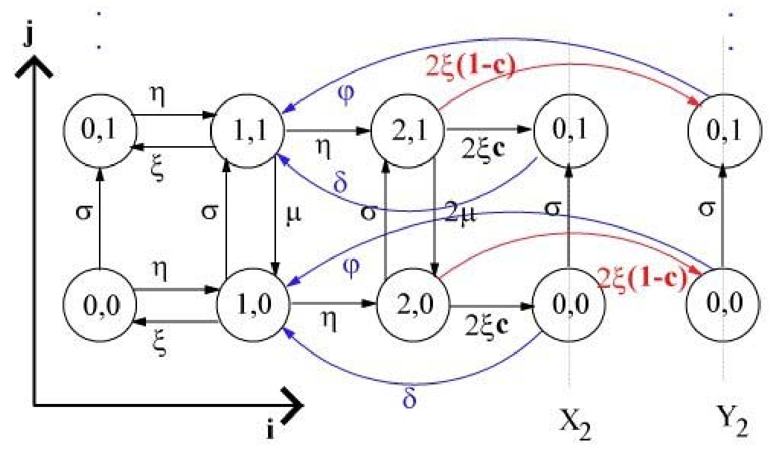
- In multiprocessor systems, some delay is encountered when
  - a failed processor is being mapped out of the system: reconfiguration/rebooting delay, and
  - when a repaired processor is being admitted into the system



Modelling ....

- The system can be represented by a QBD process with finite or infinite state space.
- The state of the system can be defined by $(I(t), J(t))$
- Define transition matrices A, A_j, B, B_j, C, C_j.

States and transitions, *K*=2



Lateral transitions, *K*=3

$$A = \begin{bmatrix} 0 & \eta & 0 & 0 & 0 & 0 & 0 & 0 \\ \xi & 0 & \eta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \eta & 2\xi c & 0 & 2\xi(1-c) & 0 \\ 0 & 0 & 0 & 0 & 0 & 3\xi c & 0 & 3\xi(1-c) \\ 0 & \delta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta & 0 & 0 & 0 & 0 & 0 \\ 0 & \varphi & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \varphi & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The steady state solution

- For an unbounded queue (i.e. $K \le L < \infty$)
- For a bounded queue (i.e. finite $L \ge K$)

State probabilities and the normalizing equation

$$\mathbf{v}_j = (p_{0,j}, p_{1,j}, \ldots, p_{N,j}); \quad j = 0,1,2,\ldots$$

$$\sum_{j=0}^{\infty} \mathbf{v}_j e = \sum_{j=0}^{\infty} \sum_{i=0}^{N} p_{i,j} = 1.0$$

Unbounded Queue – Balance Equations

$$\mathbf{v}_0\left[D_0^A + D_0^B\right] = \mathbf{v}_0 A_0 + \mathbf{v}_1 C_1$$

$$\mathbf{v}_j\left[D_j^A + D_j^B + D_j^C\right] = \mathbf{v}_{j-1} B_{j-1} + \mathbf{v}_j A_j$$

$$+ \mathbf{v}_{j+1} C_{j+1}; \quad 1 \leq j \leq M-1$$

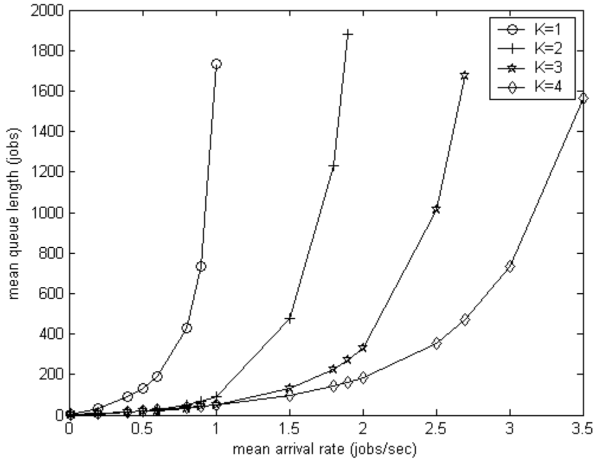
$$\mathbf{v}_j\left[D^A + D^B + D^C\right] = \mathbf{v}_{j-1} B + \mathbf{v}_j A + \mathbf{v}_{j+1} C$$

$$j \geq M$$

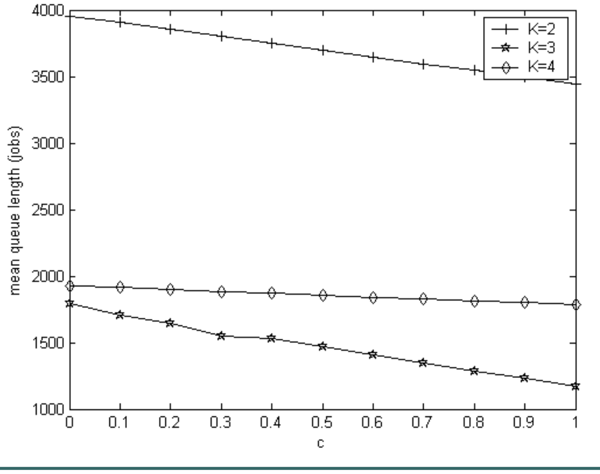
Numerical Results

- first considered 1, 2, 3, and 4-processor systems with break-downs and an infinite queue.
- Other parameters are given as s jobs/sec, ξ=0.01, η=0.5, μ=4000 jobs/hr, φ = 2 /hr, and δ = 60 /hr

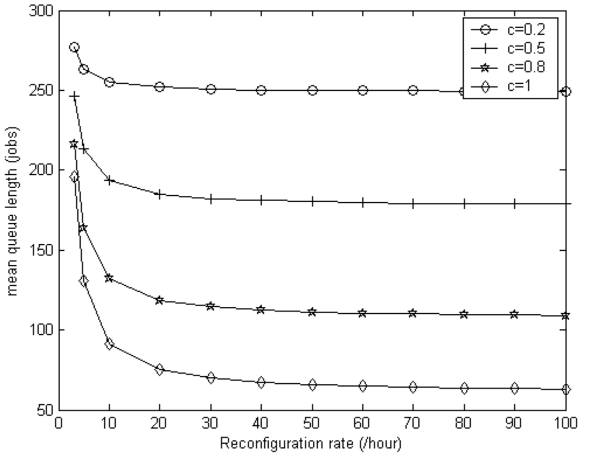
Mean Queue Length versus Mean Arrival Rate (*c* = 0)



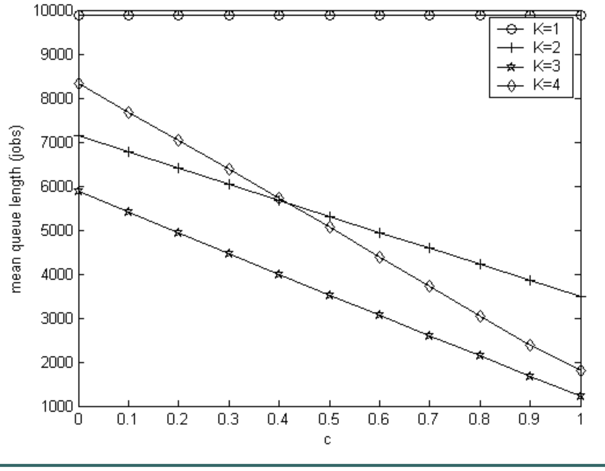
MQL vs *c* for 2, 3, and 4-Processor Systems (σ=20 jobs/sec, $\sigma/(K\mu)$=0.7, ξ=0.01, η=0.5, φ=10 /hr, and δ=50 /hr)



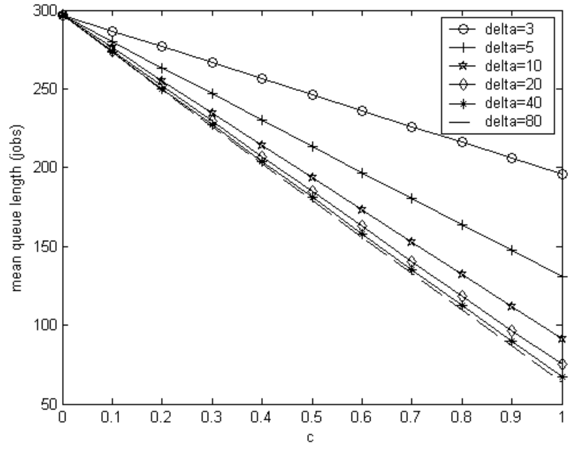
Mean Queue Length as a Function of δ and *c*



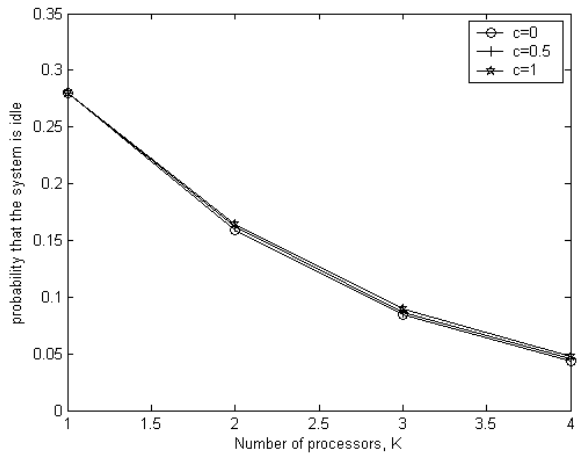
Mean Queue Length as a Function of *c*



Queue Length as a Function of c and δ



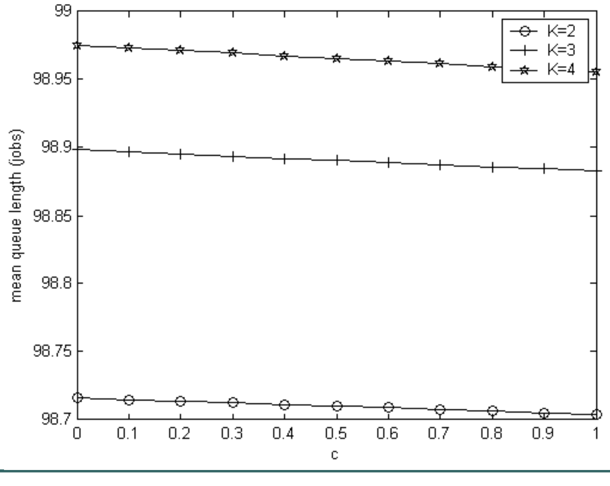
p_0 as a Function of the Number of Servers with $\sigma/(K\mu)=0.7$ and Various c Values



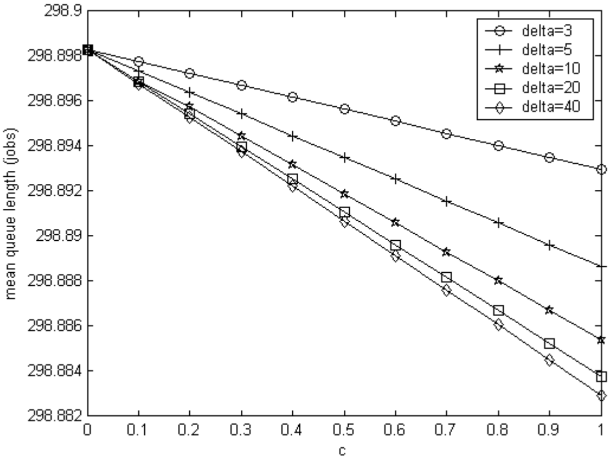
Bounded Queue

- Previous calculations were repeated for finite *L*
- Percentage of jobs lost calculated

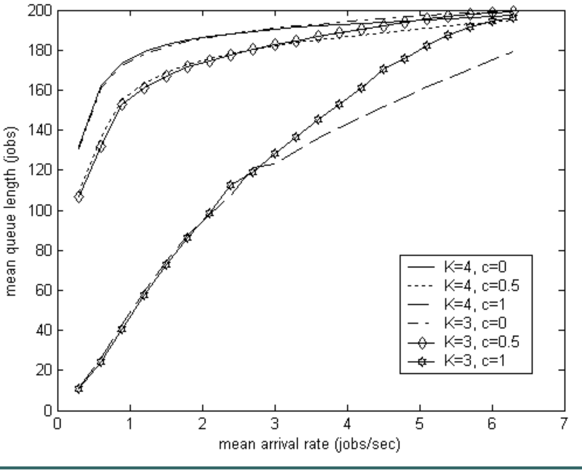
Mean Queue Length as a Function of *c* for 1, 2, 3, and 4-Processor Systems and *L* = 100



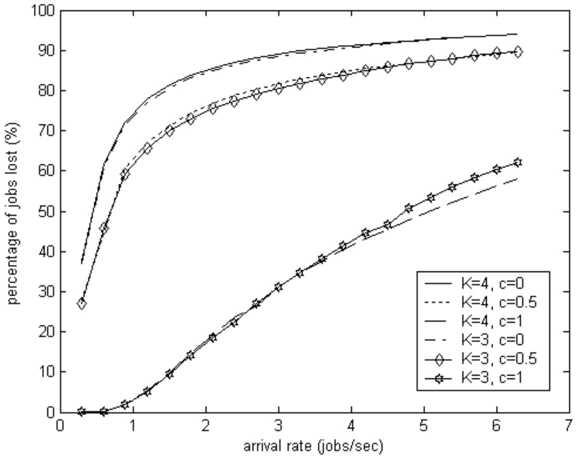
Mean Queue Length as a Function of c and d for *L* = 300



Mean Queue Length as a Function of *K*, *c*, and s for *L* = 200



Percentage of Jobs Lost as a Function of *K*, *c*, and s for *L* = 200



Conclusions

- Homogeneous multiprocessor systems with
  - break-downs, and,
  - reconfiguration and rebooting delays

  have been modelled for exact solution
- The state probabilities are derived using the spectral expansion method.
- Numerical results have been obtained and presented for various performability parameters, for both bounded and unbounded systems

Conclusions

- when queue limit is not an important factor, the choice of the optimum number of processors depends on the values of $1/\delta$, and $1/\varphi$ as well as *c.*
- *for bounded queuing systems, L is the main factor affecting the mean queue length performance of the system at relatively large σ values.*

Conclusions

- Further work
  - We have extended the work to heterogeneous microprocessor systems, with one main and a number of identical processors.
  - Work in progress.

Thanks for your attention!

Any Questions